

# On Application of Parzen-Rosenblatt Estimation for Burning Torch Studies in Infrared Band

I.A. Berg<sup>a)</sup>, S.V. Porshnev<sup>b)</sup>, A.N. Medvedev<sup>c)</sup>

*Ural Federal University, Mira 19, Yekaterinburg, Russia, 620002*

<sup>a)</sup>berg77777@gmail.com

<sup>b)</sup>s.v.porshnev@urfu.ru

<sup>c)</sup>Corresponding author: alnikmed52@gmail.com

**Abstract.** Nowadays, the fuel combustion is widely used in industry. In this context combustion processes investigation and combustion control automated systems development are the actual problems. Recently, the non-contact diagnostic methods have become widespread including the optical control in the infrared band. Analysis of the known methods of the flame scanning data processing was performed, and the result of analysis showed that known methods are useful exclusively in scientific purposes, but not under the industrial conditions. To develop the information processing methodology some experiments were carried out consisting in shooting a burning torch with a thermal imaging camera in the waves band of 1.5–5.1  $\mu\text{m}$ . During data processing, approximation of the pixels distribution over the ranges of relative temperatures was performed by the Parzen–Rosenblatt method. It was shown that calculated dependency has the same pattern on the all processed ranges. To find probability density and accumulated distribution function of abscissas of the distribution extremums, the Parzen–Rosenblatt method was used. It was proved that abscissas of the distribution extremums are stable and quasi continuous parameters, which do not depend on the time factors. Taking into account that the found parametrs can be determined in the automatic mode, it makes it possible to construct a measuring system using the infrared camera that can be placed inside the power or thermomechanical equipment and by this way construct the control and operation system of the torch combustion.

**Keywords:** thermal imager, thermograph, image processing, Parzen–Rosenblatt estimation, probability density, combustion, torch.

**PACS:** 07.05.Kf

## INTRODUCTION

Nowadays, the fuel combustion is widely used. So, the combustion processes investigation and development of advanced combustion control systems are the actual problems. Recently, the non-contact diagnostics methods have become widespread. Instead of the conventional methods [1], non-contact ones allow one to obtain the raw data more accurately and to escape impact on subject of the investigation [2].

One of the non-contact methods is the optical imaging of burning torch in the infrared band of electromagnetic radiation by the thermal imagers [3]. In this case, the radiant flux  $\Phi$  (that is detected by thermal imager) is a sum of two radiant fluxes: the self-radiation of the opacity object and part of the reflected one from the object radiation that was generated by ambient and foreign bodies

$$\Phi = \varepsilon \tau^{atm} T^n + (1 - \varepsilon) \tau^{atm} T_a^n \quad (1)$$

where  $\tau^{atm}$  is the transmitting efficiency coefficient for the current distance between the infrared camera and the object,  $T_a$  is the effective temperature of all emitting sources, which generate radiation reflected from the object and detected by the thermal imager,  $\varepsilon$  is the emissivity coefficient of the object,  $T$  is the real temperature of object. Here, index n is a constant coefficient that depends on the infrared band of imaging and technical properties of the infrared camera.

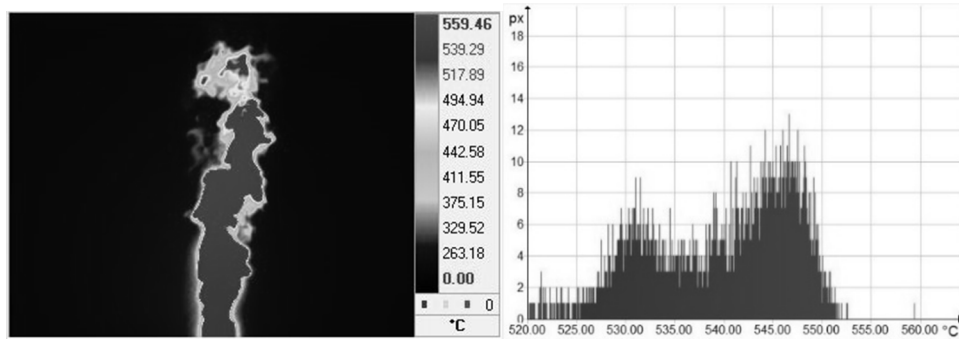
In mathematical approach, raw data at the output of the thermal imager is a 3-dimensional array of real numbers (values of the heat flux or temperature)  $T_{i,j,k}$ , where  $i, j$  are the integer numbers of the rectangular coordinates of pixels,  $i = \overline{1, I_{\max}}$ ,  $j = \overline{1, J_{\max}}$ ,  $k$  is the number of frame,  $k = \overline{1, K_{\max}}$ . Accordingly, the time series  $\tilde{T}_{I,J,k} = T_{I,J,k}$  are dependency of temperature value relayed to  $I, J$  pixel from number of frame ( $I \leq I_{\max}, J \leq J_{\max}$ ). The array of values  $\langle k\Delta t, \tilde{T}_{I,J,k} \rangle$  is dependency of temperature value relayed to  $I, J$  pixel on time; here,  $\Delta t$  is the time span between two neighbor frames. So, 2-dimensional array  $T_{i,j,K}$  represents the temperature values in plane that is orthogonal to the line passing through the center of objective lenses and focal at the instant  $K\Delta t$ ,  $K \leq K_{\max}$ .

Taking into account that the burning torch is incoherent mix of number of gases (fuel, oxygen, products of underburning, combustion products etc.) having the different concentrations, chemical composition, and structure (mono-, dual- or many atomic), the values of emissivity coefficients in different zones of torch can have significant difference. Wherein, it is not possible to determine the instantaneous concentrations of each chemical element of gases mix at each place of the torch volume.

Thus, we work under conditions when the accurate values of emissivity coefficients at each places of torch volume are still unknown. So, the task of raw data processing (obtained by thermal imaging of burning torch) completely differs from the similar tasks in nondestructive inspection field. These tasks demand new methods of the experimental data processing.

## RATIONALE FOR APPLICATION OF THE PARZEN-ROZENBLATT APPROXIMATION TO ANALYSIS OF BURNING TORCH INFRARED IMAGING DATA

Let us consider the results of torch visualization obtained during the experimental study that was performed by the automated complex for pulsation combustion investigation (Figure 1) [4, 5]. The complex consists of the vertical injection burner that generates not premixed diffusion torch and the thermal imager FLIR 7700M with the spectral range of 1.5–5.1  $\mu\text{m}$ . Imaging was performed in the windowed mode with resolution 320×256 pixels and the rate of imaging 412 Hz. Each recording was performed for 10 seconds. As a result of the thermal imaging, the frame sequences of instantaneous thermographs of burning torch were obtained. In the data processing approach, this sequences are the three-dimensional arrays  $T_{i,j,k}$ , where  $i$  is the row number,  $j$  is the column number, and  $k$  is the number of the frame; thereafter  $i_{\max} = 320$ ,  $j_{\max} = 256$ , and  $k_{\max} = 4120$ .



**FIGURE 1.** Visualization of the frame  $T_{i,j}$  from obtained frame sequence (left); distribution of the quantity of pixels at temperature ranges of this frame  $N_k(\Delta T)$  (right).

Figure 1 shows that the torch is clearly identified at this frame, because values of the pixels related to the torch are much bigger than values of ones related to the background.

Distribution of the pixels quantities over the relative temperature ranges  $N_k(\Delta T)$  for current thermograph and for general range of temperatures  $T \in [520, 570]$  is shown in Fig. 1 at right. This range of values relates to the torch pixels. Figure 1 shows that pixels distribution has two maxima in  $T \in [520, 570]$  and a local minimum between them.

For approximation of distributions of this kind, it is convenient to apply the Parzen–Rosenblatt method [6] that uses the smoothed empirical distribution function

$$F_N(y) = \frac{1}{N} \sum_{i=1}^N G\left(\frac{y-x_i}{h_N}\right) \quad (2)$$

where  $G(t)$  is the monotone nondecreasing function in the range from 0 to 1 of its argument,  $G(t) = 1 - G(-t)$ ,  $h_N$  is the smoothing parameter. After differentiation of (4), we get

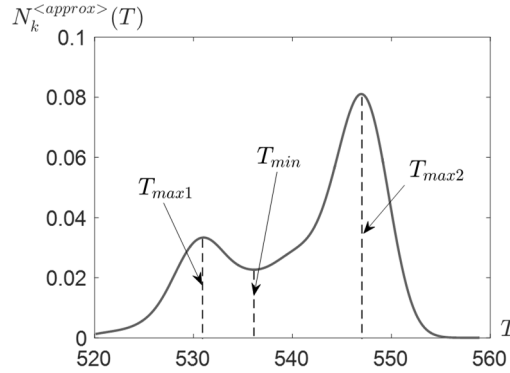
$$f_N(y) = F'_N(y) = \frac{1}{N \cdot h_N} \sum_{i=1}^N G'\left(\frac{y-x_i}{h_N}\right) = \frac{1}{N \cdot h_N} \sum_{i=1}^N K\left(\frac{y-x_i}{h_N}\right) \quad (3)$$

where  $K(t) = G'(t)$  is the probability density function of  $G(t)$  called the kernel function. In practice, the Laplace, Fisher, Cauchy, Epanechnikov, logistic, uniform, normal, triangular, or quadratic functions are usually used as the kernel functions. Reconstruction of probability density function by the Parzen–Rosenblatt method assumes solutions of two tasks: choosing the kernel function  $K(t)$  from reference list (mentioned above or others) and calculating the smoothing parameter  $h_N$ . To distinguish the best kernel function among a finite number of functions  $K(t)$ , the value of the information functional must be considered

$$J = \int \ln[K(t)]f(t)dt = \int \ln[K(t)]dF(t) \quad (4)$$

maximum value of which has to correspond to the equality condition  $K(t) = f(t)$ .

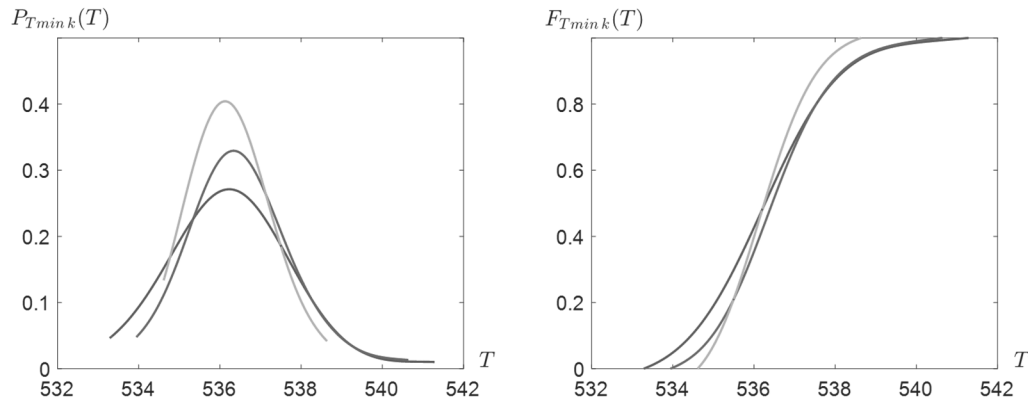
In our study, we used the software library ES&RP [7], which has software tools providing an automatic implementation of the Parzen–Rosenblatt approximation. The result of approximation  $N_k^{(approx)}(T)$  of pixels distribution from the Fig. 1 is shown in Fig. 2. Figure 2 shows that probability density of pixels  $N_k^{(approx)}(T)$  may be determined by the accurate values of each extremum's abscissa:  $T_{max1}$ ,  $T_{max2}$  and  $T_{min}$ .



**FIGURE 2.** Approximation of pixels distribution at the temperature ranges  $N_k^{(approx)}(T)$ .

Taking into account that the fuel combustion in the torch is accompanied by turbulent thermal-fluid dynamics processes, the issue of stability of selected parameters of density distribution of pixels by temperature ranges has been studied. Thereto, three time segments were selected from each of the obtained images sequences: at the beginning, in the middle, and in the end of the sequence; each segment consisted of 40 frames. Further, the Parzen–Rosenblatt approximation was performed for each frame in each segment, and then values of the abscissa extrema  $T_{max1}$ ,  $T_{max2}$ , and  $T_{min}$  were determined. As a result, for each segment of the initial frame sequence, three numerical sequences  $T_{max1k}$ ,  $T_{max2k}$ , and  $T_{min k}$  were constructed, where  $k = \overline{1, 40}$ . Then the distribution density and the distribution function of each constructed sequence were found using Parzen-Rosenblatt estimation. As an example, results of distribution density approximation of  $T_{min k}$  numerical sequences related to three selected segments is shown in Fig. 3.

Figure 3 shows that the properties of distribution density of each sequence  $T_{max1k}$ ,  $T_{max2k}$ , and  $T_{min k}$  have found to be the same. They belong to the class of functions that reach their maximum value at the considered temperature ranges at a single point. Moreover, the abscissa values corresponding to the maximum value of the distribution density of these random sequences are close to each other in each of the selected time segment. The similar picture is in the quantiles from the distribution functions.



**FIGURE 3:** Distribution densities (left) and distribution functions (right) of random variables  $T_{\min}$  determined on three time segments of the same initial frame sequence.

Thus, the selected parameters that characterize the torch combustion are turned out to be stable and quasi-stationary variables and do not depend on investigation conducted time. It should be noted that the values of these parameters of each thermograph frame of the torch burning obtained in the infrared band can be carried out in the automatic mode.

## CONCLUSION

Analysis of possibility of application of the Parzen–Rosenblatt approximation for burning torch images obtained in the infrared band processing was performed. It was showed that density of pixels quantity distribution (on temperature ranges on each frame of initial frame sequence) has stable and quasi-stationary parameters. These parameters do not depend on the influence of external random factors, but depend on the chemical composition of the fuel and of the combustion conditions. Therefore, the selected characteristics can be used to evaluate the quality of the combustion process.

Thus, it is feasible to design a non-contact monitoring system using the thermal imager. The advantage of such systems is in elimination of any impact, that measuring equipment could introduce in the process under measuring. This system can operate the fuel combustion in real time.

The described study of the torch parameters allow one to conclude that the infrared imaging snapshots can have the torch separated by temperature zones for further processing these data by the known methods for combustion simulation.

## REFERENCES

- [1] T.W. Kerlin, *Practical Thermocouple Thermometry* (International Society of Automation Research, Triangle Park, 1999).
- [2] B. Fond, "Thermographic Particle Image Velocimetry in flames: current state of the technique", Proceedings of the 18th international symposium on application of laser and imaging techniques to fluid mechanics, Springer-Verlag (2016), pp. 828-840.
- [3] A. Tommaso, M.C. Giovanni, *Infrared Thermography for Thermo-Fluid-Dynamics* (Springer-Verlag Berlin Heidelberg, Berlin, 2013).
- [4] I.A. Berg, S.V. Porshnev, B.P. Zhilkin, "Identification of pulsating combustion modes of gaseous fuel", [AIP Conference Proceedings](#) 1906, 070016 (2017); <https://doi.org/10.1063/1.5012342>.
- [5] I.A. Berg, S. V. Porshnev, V.Y. Oshchepkova, and A.N. Medvedev. "Frequency-Domain Analysis for Pulsating Combustion of Gaseous Fuel", [AIP Conference Proceedings](#) 1836, 020036 (2017); <http://doi.org/10.1063/1.4981976>.
- [6] V.N. Syzrantsev, Ya.P. Nevelev, S.L. Golofast, *Calculation of equipment durability based on the methods of the distribution-free statistics* (Science, Novosibirsk, 2008).
- [7] A.S. Koposov, S.V. Porshnev, Program library ES&RP, Certificate of state registration of computer application 2016614275 since 2016/04/20.